

2017 Raytheon MATHCOUNTS National Competition

Monday May 15, 2017 - Orlando, FL

Rank	Student	State	Grade	Rank	Student	State	Grade
1	Robitaille, Luke	TX	7	29	Verma, Rishi	OH	8
2	Cai, Andrew	TX	7	30	An, Joy	WA	6
S	Wang, William	NJ	8	31	Gu, Andrew	CA	7
S	Albright, Jack	CA	7	31	Dong, Derek	VA	8
Q	Wu, Geoffrey	IL	8	33	Chalasani, Vivek	AZ	8
Q	Choi, Reagan	MI	7	34	Ding, Jason	MO	8
Q	Xu, Alex	MI	8	35	Shi, Austin	VA	8
Q	Zhou, Jeremy	TX	8	36	Goodman, Sam	NV	7
P	Liu, Brian	NJ	8	37	Yue, William	MA	7
P	Watson, Holden	GA	8	38	Chheda, Dev	NC	8
P	Li, Kevin	CA	8	39	Jiang, Stephen	MO	8
P	Huang, Andrew	PA	8	40	Frazer, Jake	FL	7
13	Mui, Holden	IL	8	41	Goel, Gopal	OR	8
14	Tran, Coby	OR	8	42	Sharan, Vismay	FL	8
15	Chen, Alan	PA	7	43	Walsh, Noah	OR	8
16	Epstein, Ben	MA	8	44	Xu, Brian	OR	8
17	Li, David	NY	8	45	Lee, David	WA	8
18	Krishna, Chinmay	KS	8	46	Yuan, Daniel	MD	7
19	Yu, Aaron	GA	8	47	Huang, Lucas	OH	8
20	Whyte, Jaedon	FL	7	48	Zhang, Jeffrey	MI	8
21	Hu, David	CA	8	49	Yang, Eric	MA	7
22	Cong, Kevin	NJ	7	50	Goel, Abhinav Mihir	IL	8
23	Thomas, Rahul	CO	8	51	Zhou, Lawrence	GA	8
24	Xia, Daniel	NJ	7	52	Akula, Aditya	OH	8
25	Qian, Timothy	MD	8	53	Rajesh, Kishore	AZ	8
26	Hong, Daniel	WA	8	54	Florin, Sam	CT	8
27	Xiao, Justin	TX	8	55	Camacho, Joseph	NM	7
28	Deng, Timmy	NC	8	56	Yang, Kevin	IA	8

Rank	Team	Rank	Team
1	Texas	13	Kansas
2	New Jersey	14	Maryland
3	California	15	Ohio
4	Georgia	16	Colorado
5	Michigan	17	Arizona
6	Illinois	18	Missouri
7	Oregon	19	New York
8	Massachusetts	20	Connecticut
9	Florida	21	Nevada
10	Washington	22	Indiana
11	Virginia	23	Iowa
12	Pennsylvania		

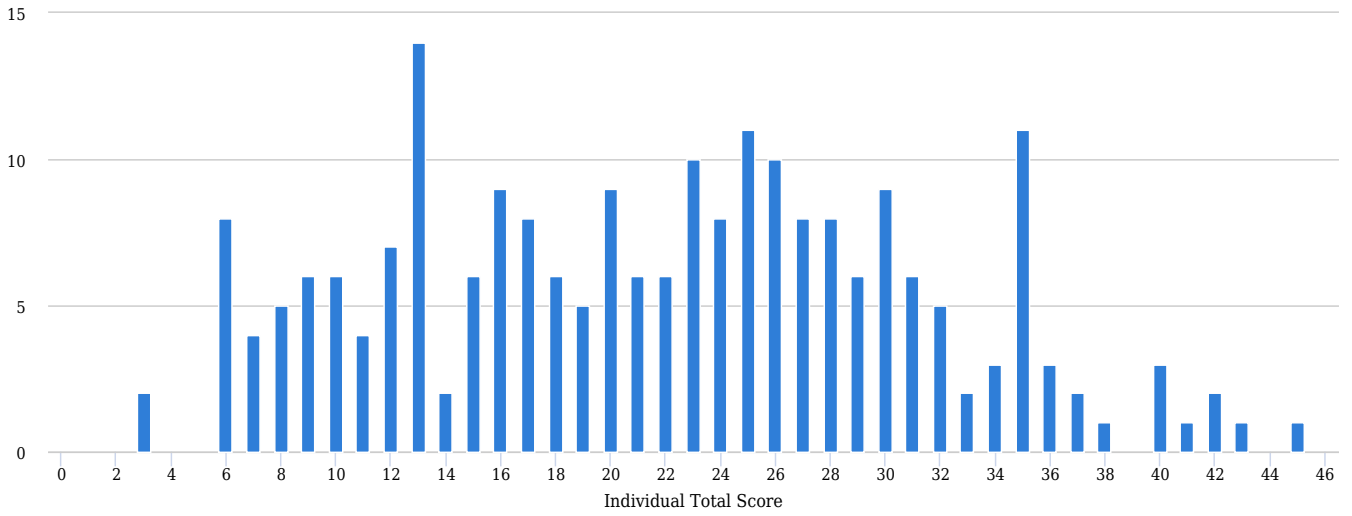
Written Competition Champion - Luke Robitaille, Texas

Written Competition Runner-Up - Geoffrey Wu, Illinois

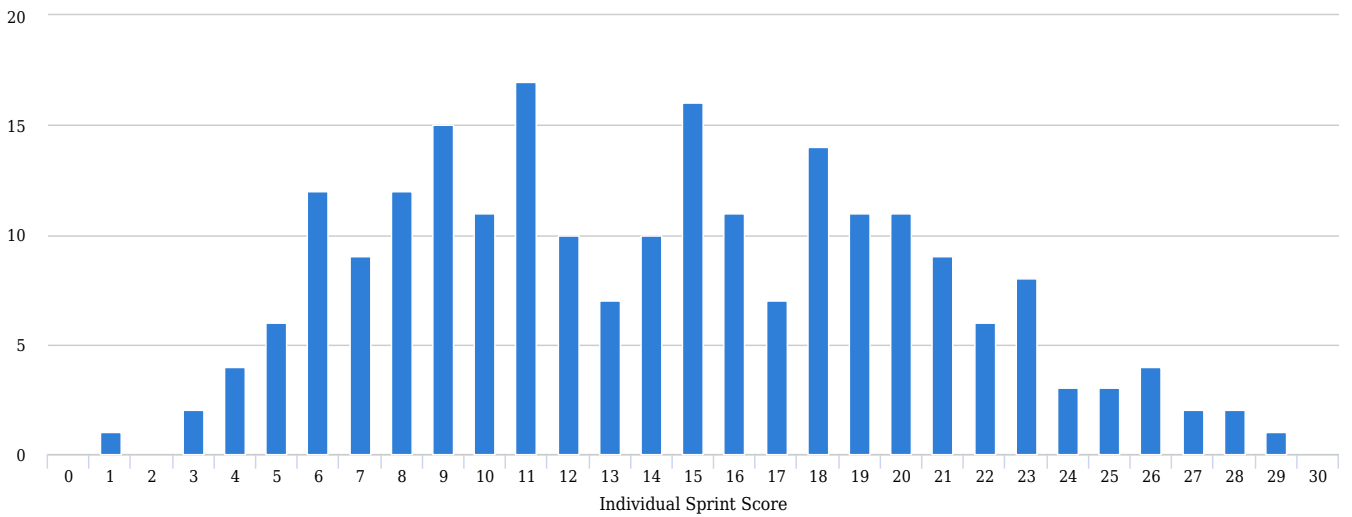
	Indiv. Total	Sprint Score	Target Score	Team Total	Team Round
Minimum	3	1	0	7.5	2
Average	21.78	14.19	7.59	34.74	12.96
Maximum	45	29	16	59.75	20
Std. Dev.	9.34	6.11	3.76	12.31	4.64

Grade	M	F	U	Total
6	11	2	0	13
7	51	15	0	66
8	127	18	0	145
U	0	0	0	0
Total	189	35	0	224

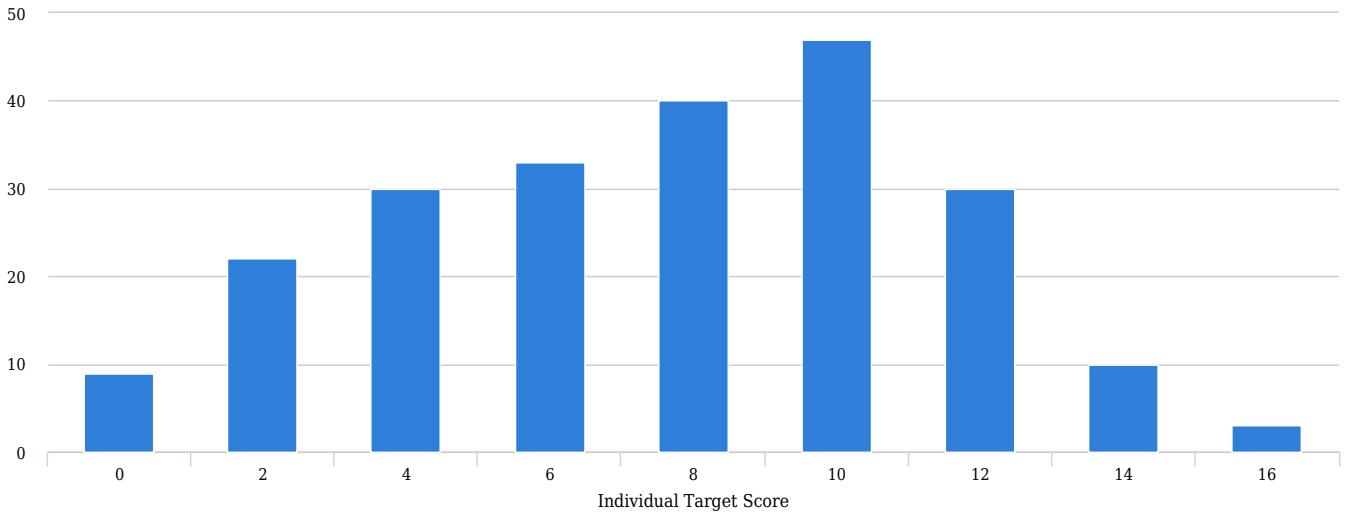
Number of Students per Individual Score



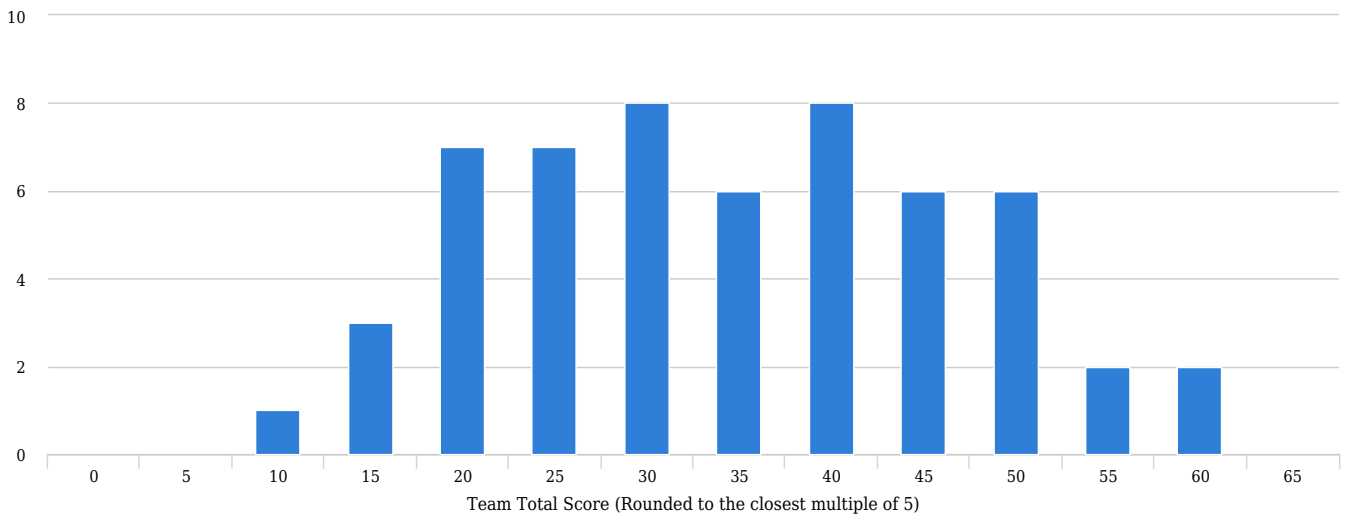
Number of Students per Sprint Score



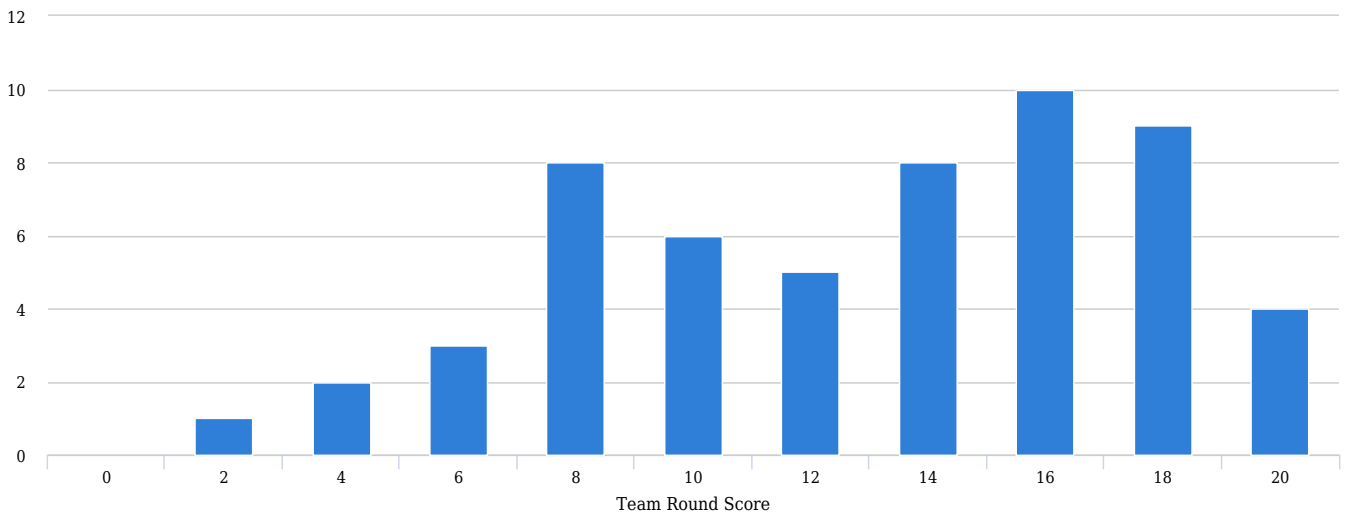
Number of Students per Target Score



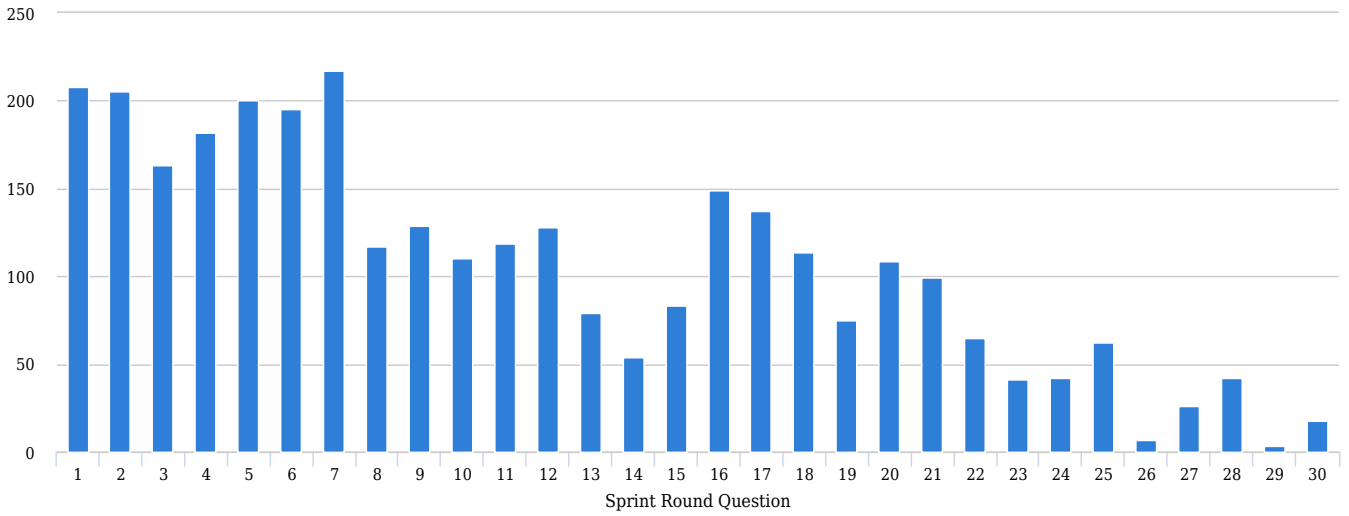
Number of Teams per Team Score



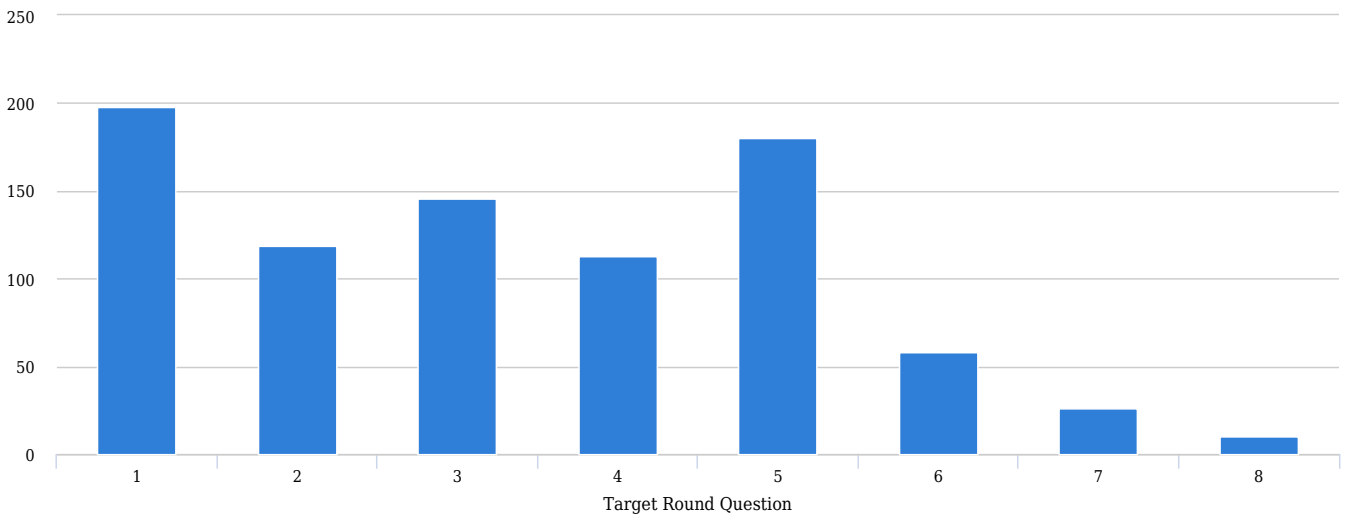
Number of Teams per Team Round Score



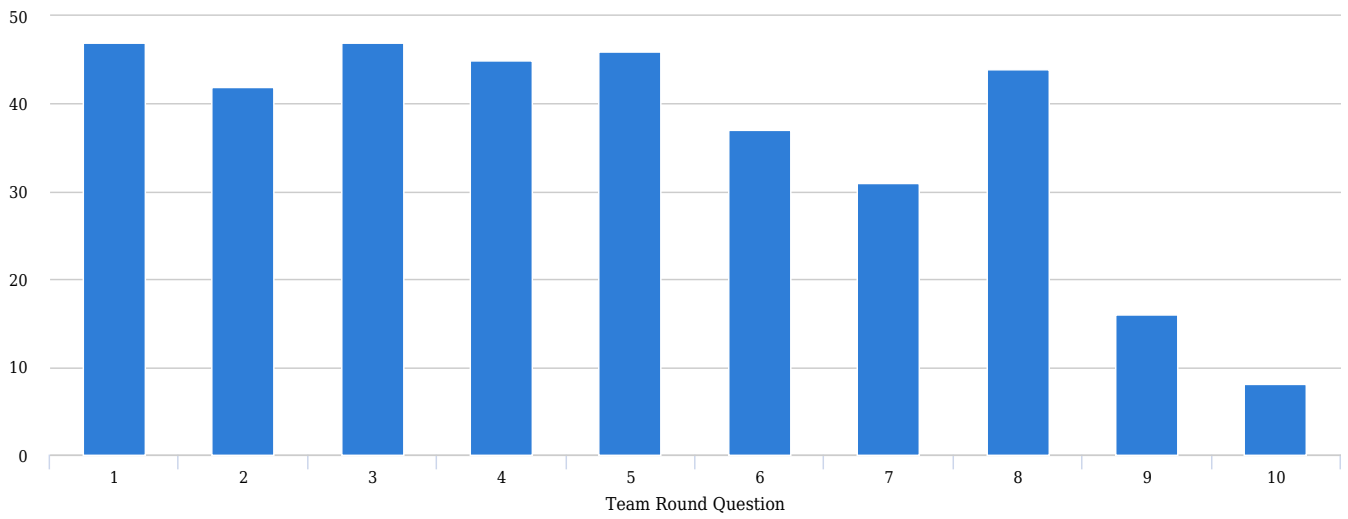
Number of Correct Responses



Number of Correct Responses



Number of Correct Responses



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Answer Key

The appropriate units (or their abbreviations) are provided in the answer blanks.

Note to coordinators: Answers to the Tiebreaker Round problems appear in the Tiebreaker Round Booklet.

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Sprint Round Answers

- | | | |
|---------------------------|------------------------|-------------------------------------|
| 1. 5600 gallons | 11. 1 | 21. 22 |
| 2. 72 | 12. 8 gallons | 22. 220 ways |
| 3. 44 percent | 13. $8\sqrt{10}$ units | 23. $2\sqrt{3}$ cm ² |
| 4. 144 units ² | 14. $\frac{9}{64}$ | 24. $\frac{5}{16}$ |
| 5. $-\frac{6}{5}$ | 15. 550 | 25. 10 units |
| 6. 672 terms | 16. 29 students | 26. $\frac{5}{23}$ |
| 7. -4 | 17. $12\sqrt{3}$ cm | 27. $9\sqrt{19}$ units ² |
| 8. 16 base 8 | 18. $\frac{9}{16}$ | 28. 0.25 |
| 9. 11 integers | 19. $\frac{5}{16}$ | 29. 90.8 inches |
| 10. 72 squares | 20. 35,287 | 30. $2 - \sqrt{3}$ units |

IMPORTANT NOTICE REGARDING SPRINT #25

The intended interpretation for Sprint Round 25 is that side AB has length 9 units, side BC has length 10 units and side AC has length 13 units. The correct answer, given these assigned side lengths, is 10 units. Two alternate answers result if side lengths are assigned differently. Due to this ambiguity, all three answers were accepted.

Target Round Answers

- | | | | |
|---------------|--------------------|--------|--|
| 1. 49 | 3. 67,800,320 ways | 5. 23 | 7. $\sqrt{42} - 5$ or $-5 + \sqrt{42}$ |
| 2. 61 percent | 4. 72 degrees | 6. -17 | 8. 288 cryptocodes |

Team Round Answers

- | | |
|--------------------|---------------|
| 1. 4 prime numbers | 6. 21 ways |
| 2. 12 lines | 7. 55 |
| 3. 444 | 8. 21 |
| 4. $\frac{1}{4}$ | 9. 165 meters |
| 5. 40 mi/h | 10. 3420 ways |

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Sprint Round
Problems 1–30

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

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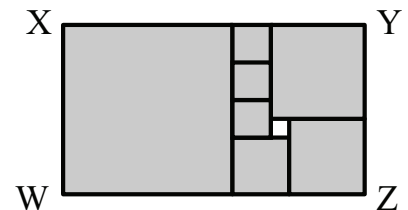
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1. _____ gallons The gauge on an oil tank indicated that the tank was two-sevenths full. After 3200 gallons were added to the tank the gauge indicated that the tank was six-sevenths full. How many gallons of oil will the tank hold when full?

2. _____ What is the maximum possible absolute difference between a two-digit integer and the two-digit integer resulting when the digits are reversed?

3. _____ percent At Salisbury Central School, 60% of the students are 7th graders, and the rest are 8th graders. Half of the 7th graders ride a bus to school, 20% of the remaining 7th graders arrive by car, and the rest walk to school. A quarter of the 8th graders ride a bus to school, a third of the remaining 8th graders arrive by car, and the rest walk to school. What percent of Salisbury Central School students walk to school?

4. _____ units² Rectangle WXYZ, shown here, consists of eight squares. If the white square is a 1×1 unit square, what is the area of rectangle WXYZ?



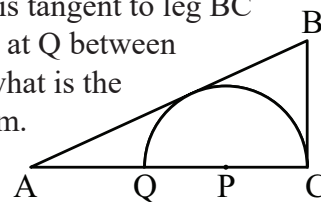
5. _____ What is the slope of a line perpendicular to the line given by the equation $\frac{x-8}{y+4} = \frac{6}{5}$? Express your answer as a common fraction

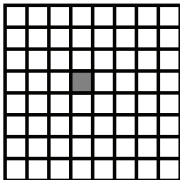
6. _____ terms In the sequence 121, 1221, 12221, ... the n th number consists of n copies of the digit 2, surrounded by two 1s. How many of the first 2017 terms in the sequence are divisible by 3?
7. _____ For what value of x is $|-x + |x| + 1|$ equal to 9?
8. _____ base 8 Computing in base 8, a certain two-digit base-8 number N is added to five times the sum of its digits. The sum has the same digits as N but in reverse order. What is N in base 8?
9. _____ integers The median of a list of positive integers is 3, and the mean of the list is less than 2.1. What is the fewest number of integers possible in the list?
10. _____ squares How many perfect squares are divisors of the product $1! \cdot 2! \cdot 3! \cdot 4! \cdot 5! \cdot 6! \cdot 7!$?

11. _____ If x and y are nonzero real numbers such that $mx + ny = u$, $nx - my = v$ and $u^2 + v^2 = x^2 + y^2$, what is the value of $m^2 + n^2$?

12. _____ gallons Zeno's tank holds 24 gallons of water. Zeno starts filling the tank, but when it is halfway full he decides to start draining water out. Once half of the water that he added is drained, he decides to add back half of the water that he just drained. He then drains half of the water that he just added, and continues alternately adding or draining half of the previous quantity of water. After 100 cycles of adding and draining water, how many gallons of water are in the tank? Express your answer to the nearest whole number.

13. _____ units In right triangle ABC with right angle at vertex C , a semicircle is constructed, as shown, with center P on leg AC , so that the semicircle is tangent to leg BC at C , tangent to the hypotenuse AB , and intersects leg AC at Q between A and C . The ratio of AQ to QC is $2:3$. If $BC = 12$, then what is the value of AC ? Express your answer in simplest radical form.



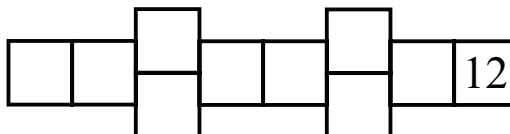
14. _____  Philippa stands on the shaded square of the 8-by-8 checkerboard shown. She moves to one of the four adjacent squares sharing an edge with her starting square, with each of the four squares equally likely to be chosen. She then makes two more moves to adjacent squares in the same way. Given any square S , let $P(S)$ be the probability that Philippa lands on that square after her third move. What is the greatest possible value of $P(S)$? Express your answer as a common fraction.

15. _____ If n is a positive integer and D is a digit such that $\frac{n}{814} = 0.\overline{D75}$, what is the value of n ?

16. _____ students A college dorm houses math majors and chemistry majors. There is at least one student in each major and no students are majoring in both. Students live in single or double rooms. If four-fifths of the math majors are roommates with six-sevenths of the chemistry majors, what is the least possible number of students living in the dorm?
17. _____ cm The sum of twelve times the numerical value of the total length, in centimeters, of the edges of a cube and the numerical value of its volume, in cubic centimeters, is equal to four times the numerical value of the total surface area, in square centimeters. What is the length of its space diagonal? Express your answer in simplest radical form.
18. _____ The diagonals of parallelogram ABCD intersect at E. Point F is the midpoint of segment BE and H is the midpoint of segment CE. What is the ratio of the area of quadrilateral AFHD to the area of the parallelogram? Express your answer as a common fraction.
19. _____ Sam creates a six-digit positive integer by writing the digit 7 in the hundred-thousands place, and then tossing a fair coin five times. If the coin comes up heads, he writes a 7 for the next digit; if the coin comes up tails, he writes a 0 for the next digit. What is the probability that Sam's number is divisible by 77? Express your answer as a common fraction.
20. _____ What is the sum of the positive integers less than 1000 that are multiples of 7 but not multiples of 2?

21. _____ Suppose f is a quadratic function defined by $f(x) = ax^2 + bx + c$ for some numbers a , b and c . If $g(x) = x - 2$ and $f(g(x)) = 2x^2 - 5x + 19$ for all values of x , what is the value of $a + b + c$?

22. _____ ways How many ways are there to fill in each empty square in the diagram below with a positive integer so that no integer appears more than once in the diagram, and every integer in the diagram is less than each integer to its right?



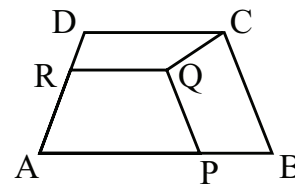
23. _____ cm^2 What is the total surface area of the largest regular tetrahedron that can be inscribed inside of a cube of edge length 1 cm? Express your answer in simplest radical form.

24. _____ Penny flips three fair coins into a box with two compartments. Each compartment is equally likely to receive each of the coins. What is the probability that either of the compartments has at least two coins that landed heads? Express your answer as a common fraction.

25. _____ units Triangle ABC has side lengths 9, 10 and 13, with D the midpoint of side BC. What is the length of segment AD?

26. _____ In a pile of 25 tiles, each tile has one of the letters A through E and one of the integers from 1 through 5. Each possible combination of a letter and a number appears on exactly one of the tiles. Jessica selects three tiles at random, without replacement, from the pile. What is the probability that each of the three tiles Jessica chooses has a letter or a number in common with at least one of the other chosen tiles? Express your answer as a common fraction.

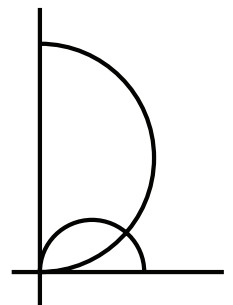
27. _____ units² In isosceles trapezoid ABCD, shown here, sides AB and DC are parallel, AB = 10 and CD = 8. Trapezoids APQR and BCQP are both similar to trapezoid ABCD. What is the area of trapezoid ABCD? Express your answer in simplest radical form.



28. _____ What is the value of $2\sqrt{1.75 + \sqrt{3}} - (1.75 + \sqrt{3})$? Express your answer as a decimal to the nearest hundredth.

29. _____ inches A 36-inch rope is cut into three pieces. One piece is five inches longer than another, and one piece is twice as long as another. What is the sum of the possible lengths of the longest piece? Express your answer as a decimal to the nearest tenth.

30. _____ units In the figure shown, two lines intersect at a right angle, and two semicircles are drawn so that each semicircle has its diameter on one line and is tangent to the other line. The larger semicircle has radius 1. The smaller semicircle intersects the larger semicircle, dividing the larger semicircular arc in the ratio 1:5. What is the radius of the smaller semicircle? Express your answer in simplest radical form.



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Target Round
Problems 1–8

Name _____

State _____

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This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the problem sheets. If you complete the problems before time is called, use the time remaining to check your answers.

Total Correct	Scorer's Initials

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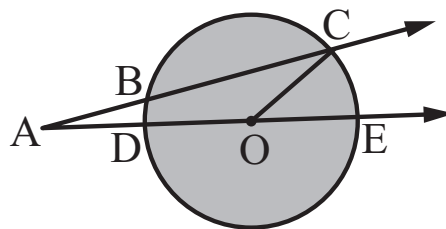
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1. _____ Let x , y and z be consecutive integers such that $x < y < z$. If $(x + y)(x + z) = 9900$, what is the value of x ?

2. _____ percent This year, the city coed softball league has 15% more participants than it had last year. There are 8% more male participants and 20% more female participants than last year. What percent of the league's participants are female this year? Express your answer to the nearest whole number.

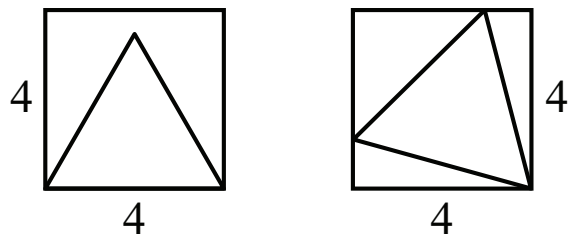
3. ways There are a hundred competitors at the National Debating Contest, two from each of the 50 states. In how many ways can five finalists be chosen if no state may have more than one finalist?

4. degrees Rays AC and AE intersect circle O at B and D, respectively. Segment DE is a diameter of circle O and $AB = \frac{1}{2} DE$. If the measure of $\angle BAD$ is 24 degrees, what is the degree measure of $\angle COE$?



5. _____ A set of five distinct prime numbers has a mean of 10 and a median of 7. What is the greatest possible number in this set?

6. _____ Two congruent squares with side length 4 have equilateral triangles constructed in them as shown. In one square, one side of the equilateral triangle is a side of the square. In the other square, the equilateral triangle has one vertex at a vertex of the square and its other two vertices are on the sides of the square. The absolute difference of the areas of the two triangles can be expressed in simplest radical form as $a\sqrt{b} + c$. What is the value of $a + b + c$?



7. _____ The graph of the equation $\frac{xy - y}{2x - y + 2} = a$, where a is a positive number, intersects the line $y = x$ at two points that are a distance of exactly 6 units apart. What is the value of a ? Express your answer in simplest radical form.

8. _____ crypto-
codes A certain cryptocode must contain one letter from the set $\{X, K, M, Z\}$ and three distinct letters from the set $\{W, X, Y, Z\}$. The four letters can be arranged in any order, and since X and Z are in both sets, these letters may each appear twice in an arrangement. How many cryptocodes are possible?

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Team Round
Problems 1–10

State _____

Team
Members _____, Captain

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This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

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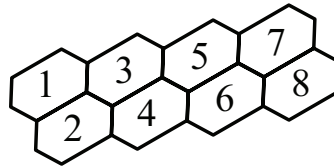
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1. prime numbers For how many two-digit prime numbers p does swapping the digits of p produce a prime number greater than p ?
2. lines There are infinitely many lines that are perpendicular to the line $y = \frac{3}{2}x + 9$ and intersect it in the interior of the second quadrant of the coordinate plane. How many of these lines have integer y -intercepts?
3. _____ Madison writes a one-digit positive integer, a two-digit positive integer, and a three-digit positive integer with the digits 1 through 6, using each digit exactly once. The product of the three positive integers is 20,400. What is the sum of the three positive integers?
4. _____ A strip of uniform width is cut from three sides of a square. The area of the remaining rectangle is $\frac{3}{8}$ of the area of the original square. What is the ratio of the width of the uniform strip to the side length of the original square? Express your answer as a common fraction.
5. mi/h Bebe used three forms of transportation to get from Portland to Anchorage. She traveled 150 miles by train. She traveled 80 miles by bus at an average speed that was 10 mi/h less than that of the train. She traveled 1500 miles by plane at an average speed that was 10 times that of the train. If these three portions of the trip took a total of 8 hours, what was the average speed of the bus?

6. _____ ways In a game, eight hexagons are arranged as shown. Starting at hexagon 1, a path to hexagon 8 is created by moving to an adjacent hexagon whose value is greater than the preceding hexagon. Two hexagons are considered adjacent if they share a side. How many ways are there to get from hexagon 1 to hexagon 8?



7. _____ After their final practice, three math teams celebrated by ordering hot dogs and potato patties. The Thales team ordered ten of each and paid \$63.00. The Euclid team ordered five potato patties and twelve hot dogs for \$46.55. The total price the Archimedes team paid was a whole number of dollars, and this number was a palindrome. What is the least number that palindrome could have been?
8. _____ The sum of the reciprocals of four different positive integers is 1.9. What is the sum of the four integers?
9. _____ meters Two leopard seals, Snap and Snarl, start 210 meters apart. They swim toward each other at a constant speed of 10 km/h each. Gilly, a gentoo penguin, starts at Snap and swims back and forth between the seals continually until the two seals meet. When going from Snap to Snarl, Gilly swims at 15 km/h, but when going from Snarl to Snap, Gilly swims at 20 km/h. What is the total distance that Gilly swims before the seals meet?
10. _____ ways Jay has seven different cars that he is leaving to his three daughters and two sons. The Maserati must go to a daughter, and the Bentley must go to a son. Each heir is to receive at least one and no more than two cars. How many ways can the cars be distributed?

